Musings on the BES Critical Point Search

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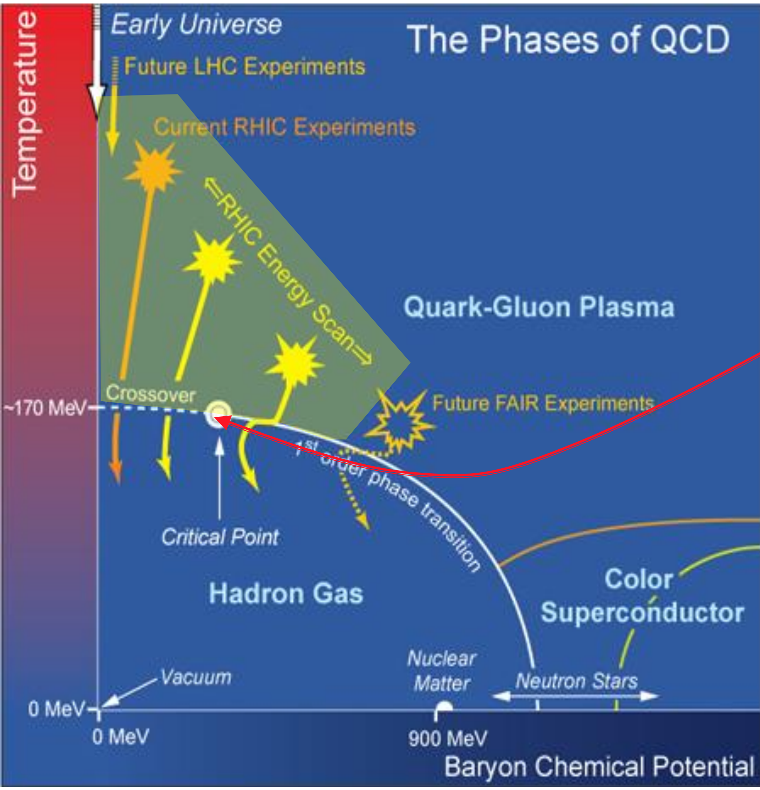
Outline

- ***Introduction***
  - *Questions*
  - *Known's & unknowns*
- ***Search strategy***
  - *Guiding principles*
  - *Basics of Finite-Time Effects*
  - *Basics of Finite-Size Effects*
- ***Probes & Characterizing the CEP***
  - *HBT*
  - *Finite-Time-Scaling*
  - *Finite-Size-Scaling*
- ***Summary***
  - ***Epilogue***

The QCD Phase Diagram



Essential Question

What ingredients are required to fully characterize *the CEP "landmark"*?

- *Its location* (T^{cep}, μ_B^{cep})*?*
- *Its static critical exponents -* ν, γ*?*
  - ✓ *Static universality class?*
  - ✓ *Order of the transition*
- *Dynamic critical exponent/s – z?*
  - ✓ *Dynamic universality class?*

All are required to fully characterize the CEP

- *Validation of the first order phase transition → added bonus*

Knowns & unknowns

Known known

Theory consensus on the static universality class for the CEP

3D-Ising Z(2), $\nu \sim 0.63$, $\gamma \sim 1.2$

Summary - M. A. Stephanov Int. J. Mod. Phys. A 20, 4387 (2005)

Recent experimental Validation

Lacey, PRL 114 (2015),142301

Known unknowns

- **Location (T^{cep}, μ_B^{cep}) of the CEP?**

  Summary - M. A. Stephanov Int. J. Mod. Phys. A 20, 4387 (2005)

- ***Dynamic Universality class for the CEP?***
  - ✓ **One slow mode (L),** ***z ~ 3 - Model H***

    Son & Stephanov, Phys.Rev. D70 (2004) 056001
    Moore & Saremi, JHEP 0809, 015 (2008)

  - ✓ **Three slow modes (NL)**
    - ✓ $z_T \sim 3$
    - ✓ $z_v \sim 2$

      } ***[critical slowing down]***
    - ✓ $z_s \sim -0.8$ ***[critical speeding-up]***

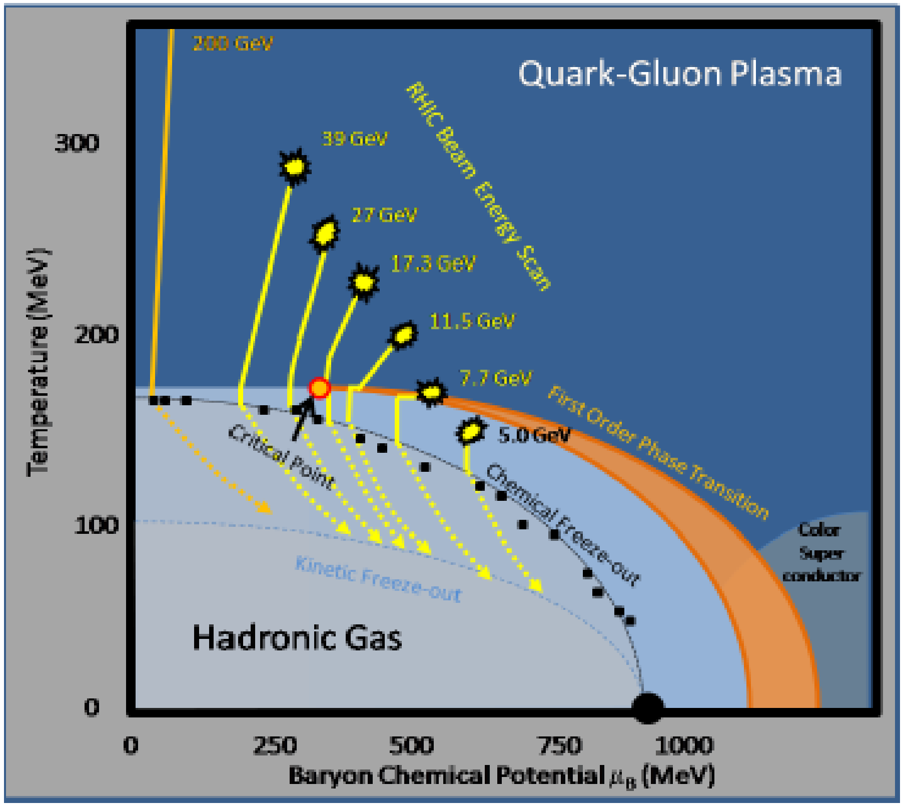
    Y. Minami **- Phys.Rev. D83 (2011) 094019**

Knowledge about the dynamic critical exponent/s is crucial

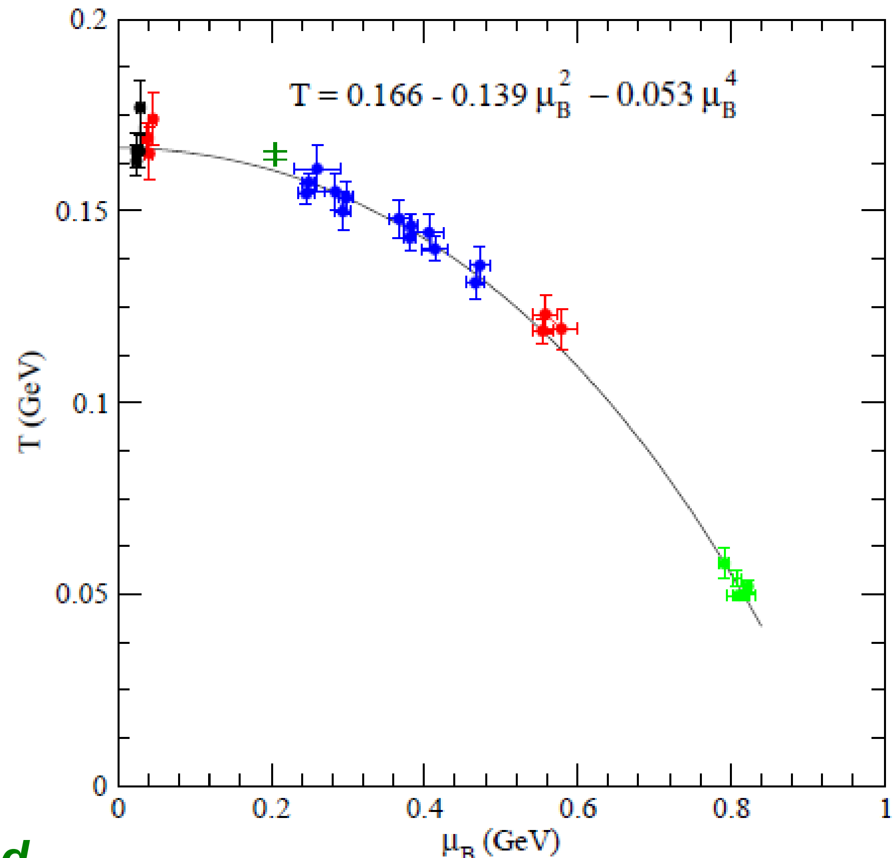
Experimental verification and characterization of the CEP is an imperative

Ongoing beam energy scans to probe a large ***(T,μ_B) domain***

(T,μ_B) –Domain



(μ_B, T) at chemical freeze-out (CFO)



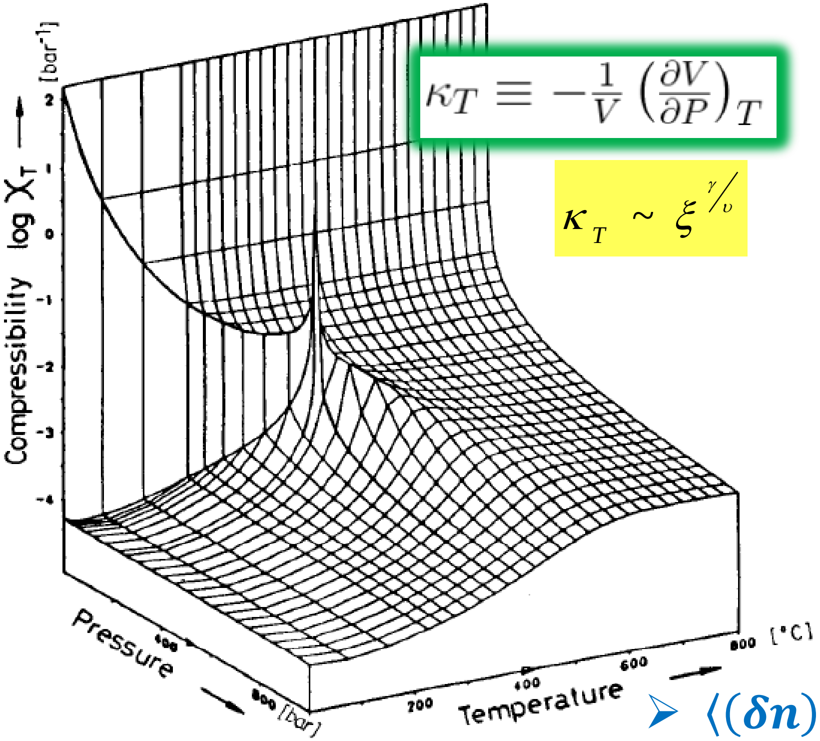
- ***LHC → access to high T and small μ_B***
- ***RHIC → access to different systems and a broad domain of the (μ_B,T)-plane***

$RHIC_{BES}$ to LHC → ~360 $\sqrt{s_{NN}}$ increase

$\sqrt{s_{NN}}$ is a good proxy for exploring the (T, μ_B) plane for experimental signatures → especially important if CFO is close to the phase boundary

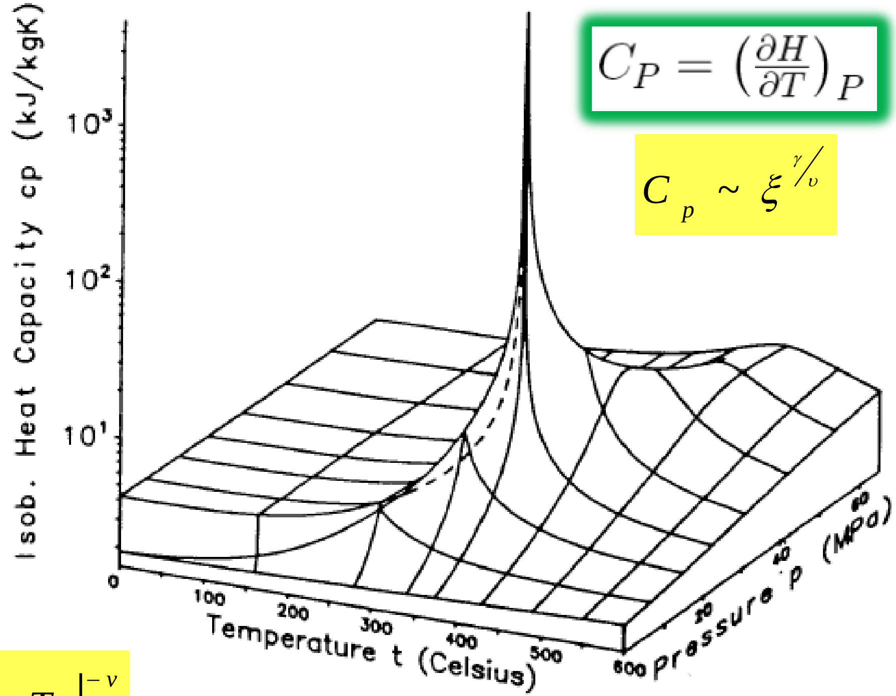
Anatomy of search strategy

H_2O



$$\kappa_T \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\kappa_T \sim \xi^{\gamma/\nu}$$



$$C_P = \left(\frac{\partial H}{\partial T}\right)_P$$

$$C_p \sim \xi^{\gamma/\nu}$$

$$\xi \sim |T - T_c|^{-\nu}$$

➢ $\langle(\delta n)\rangle \sim \xi^2$ → ***search for "critical" fluctuations in HIC***

Stephanov, Rajagopal, Shuryak, PRL.81, 4816 (98)

The critical point is characterized by several (power law) divergences linked to the correlation length ξ

<u>Central idea</u> → use beam energy scans to vary μ_B & *T to search for the influence of such divergences!*

Finite size/time effects significantly dampen these divergences → non-monotonic behavior

Basics of Finite-Time Effects

χ_{op} diverges at the CEP
so relaxation of the order parameter could be anomalously slow

$\tau \sim \xi^{z}$ ← dynamic critical exponent

Non-linear dynamics →
Multiple slow modes

*$z_T \sim 3$, $z_v \sim 2$, **$z_s \sim -0.8$***

$z_s < 0$ - Critical speeding up
$z > 0$ - Critical slowing down

Y. Minami - **Phys.Rev. D83 (2011) 094019**

An important consequence

$$\xi \sim \tau^{1/z}$$

Significant signal attenuation for short-lived processes with $z_T \sim 3$ or $z_v \sim 2$

eg. $\langle(\delta n)\rangle \sim \xi^2$ **(without FTE)**

$\langle(\delta n)\rangle \sim \tau^{1/z} \ll \xi^2$ **(with FTE)**

****Note that observables driven by the sound mode would <u>NOT</u> be similarly attenuated****

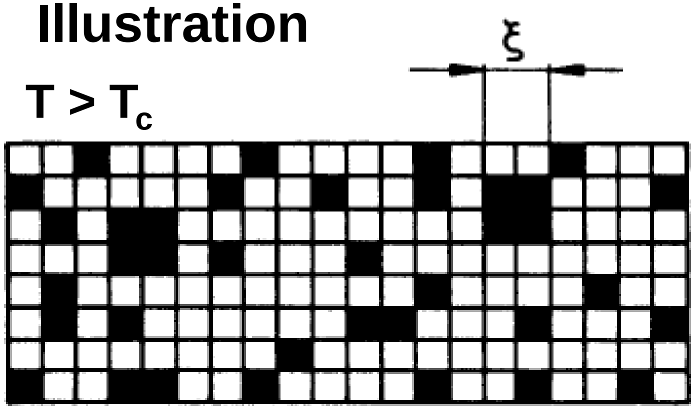
The value of the dynamic critical exponent/s is crucial for HIC

Dynamic Finite-Size Scaling (DFSS) can be used to estimate the dynamic critical exponent z
→ employed in this study

Basics of Finite-Size Effects

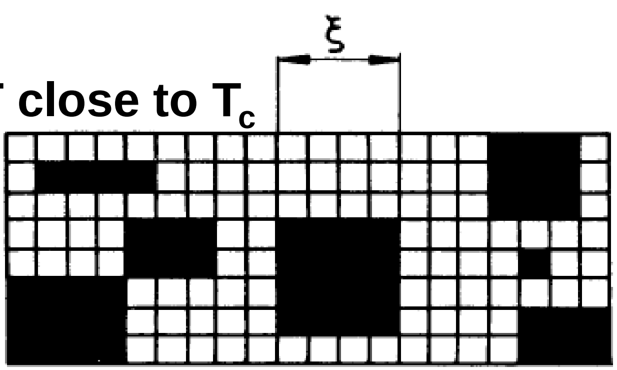
Illustration

T > T_c

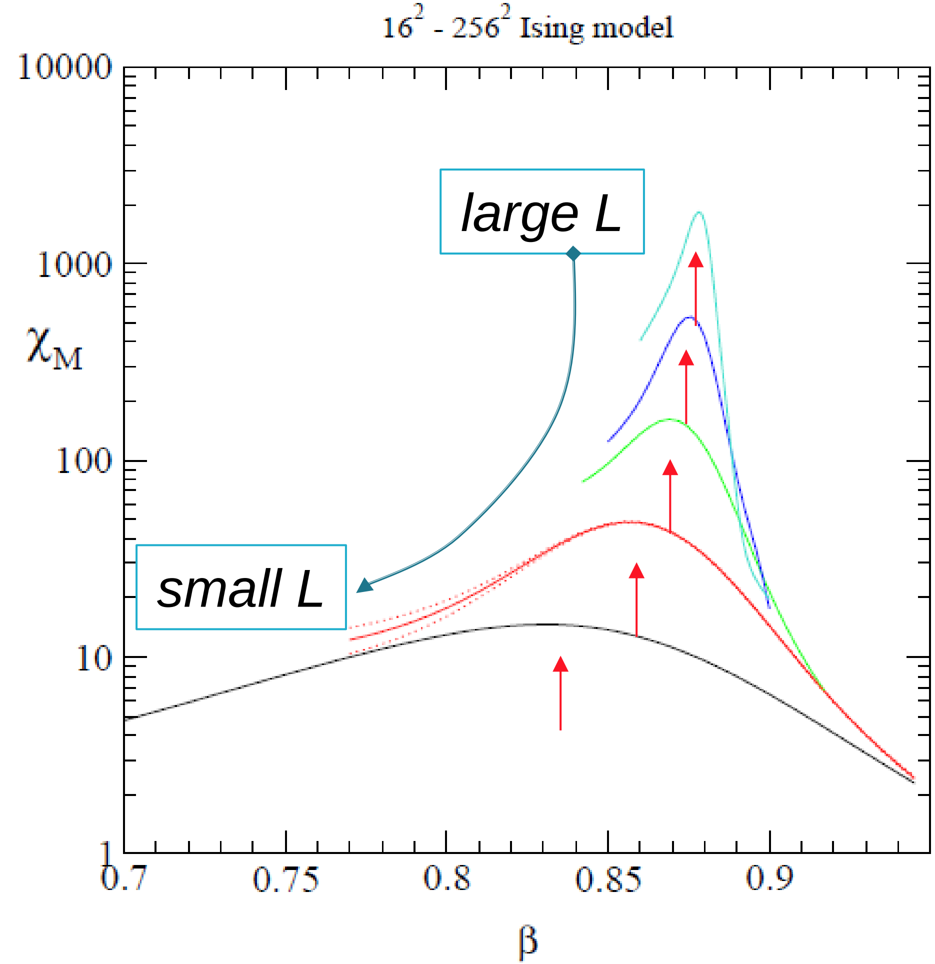


L characterizes the system size

T close to T_c



$$\xi \sim |T - T_c|^{-\nu} \leq L$$



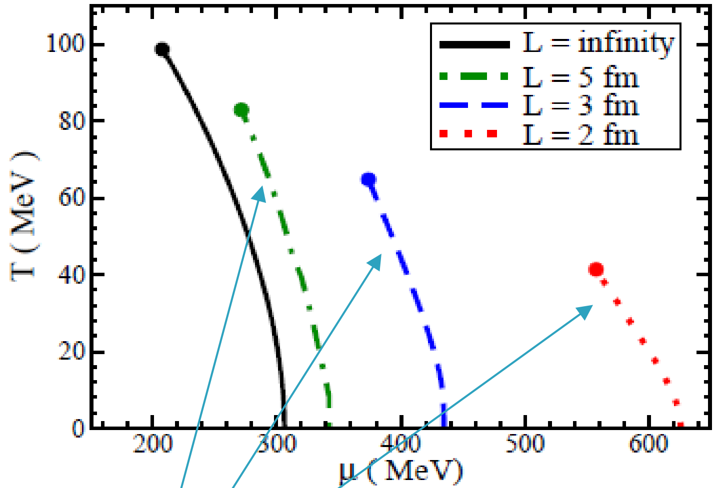
note change in peak heights, positions & widths

→ A curse of Finite-Size Effects (FSE)

→ Only a pseudo-critical point is observed → shifted from the genuine CEP

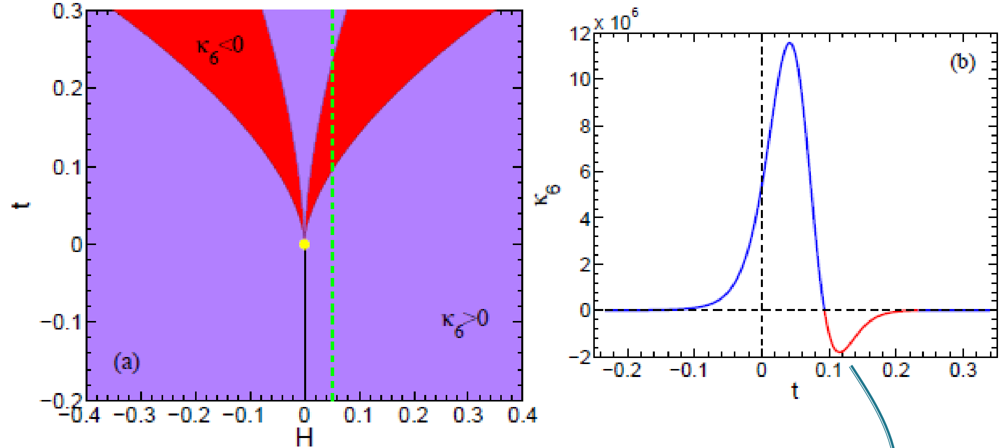
The curse of Finite-Size effects

E. Fraga et. al.
J. Phys.G 38:085101, 2011

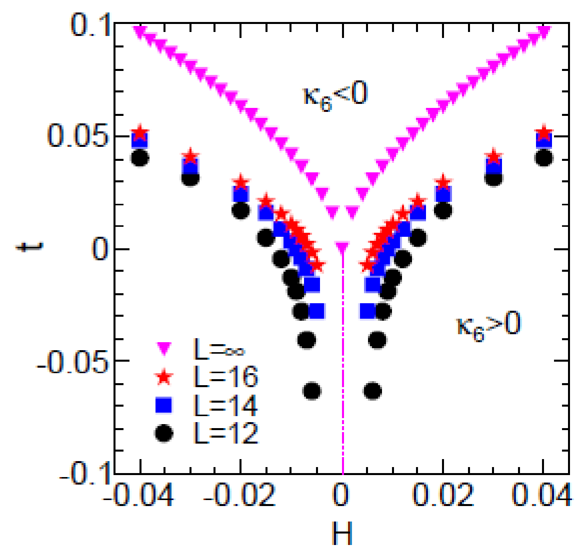


Displacement of pseudo-first-order transition lines and CEP due to finite-size

Finite-size effects on the sixth order cumulant -- 3D Ising



Pan Xue et al arXiv:1604.06858



FSE on temp dependence of minimum

FSE ~ $L^{2.5n}$ for Z(2)

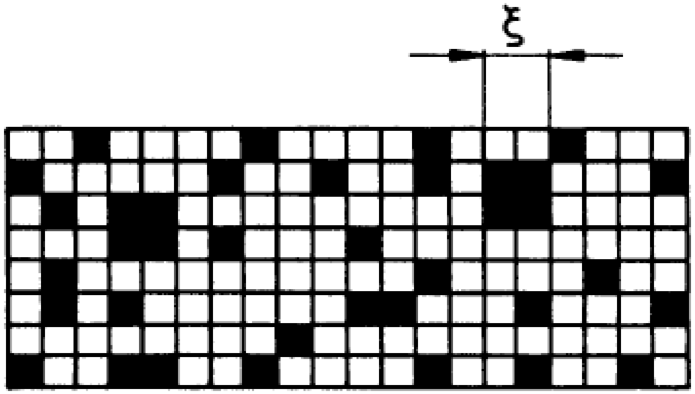
*A flawless measurement, sensitive to FSE, **can <u>not</u>** be used to locate and characterize the CEP directly*

The Blessings of Finite-Size

$$\chi_T^{\max}(V) \sim L^{\gamma/\nu},$$
$$\delta T(V) \sim L^{-\frac{1}{\nu}},$$
$$\tau_T(V) \sim T^{\rm cep}(V) - T^{\rm cep}(\infty) \sim L^{-\frac{1}{\nu}},$$

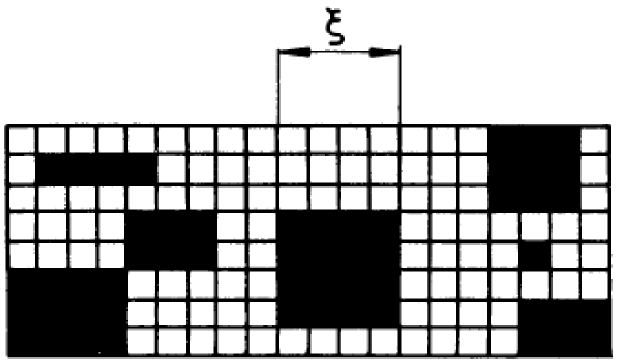
$$\chi(T, L) = L^{\gamma/\nu} P_\chi(tL^{1/\nu}) \qquad t = (T - T_c)/T_c$$

M. Suzuki, Prog. Theor. Phys. 58, 1142, 1977



a) $T > T_c$

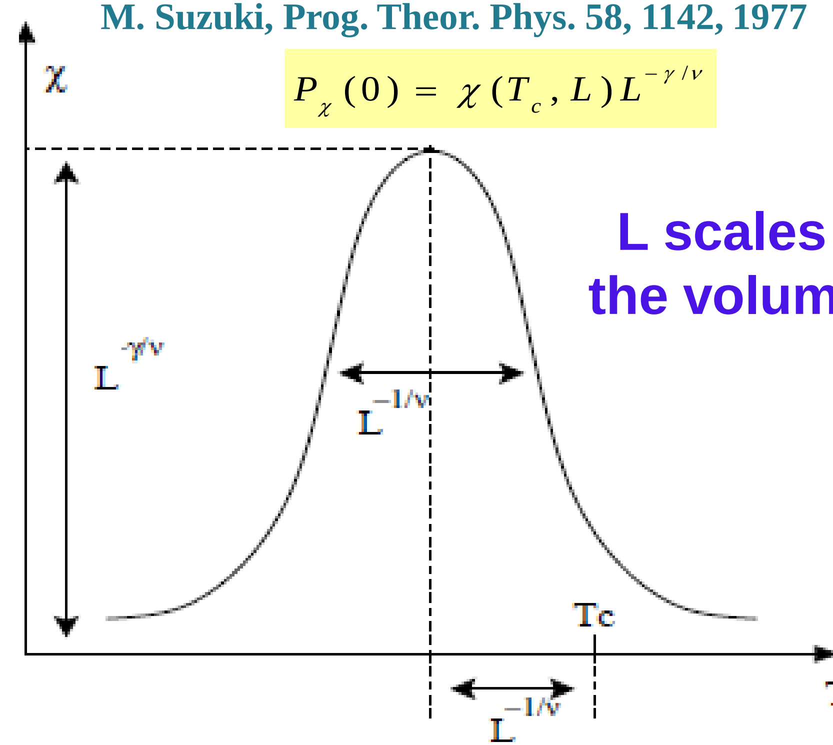
Finite-size effects have a specific L dependence



b) T close to T_c

$$\xi \sim |T - T_c|^{-\nu} \leq L$$

$$P_\chi(0) = \chi(T_c, L) L^{-\gamma/\nu}$$



L scales the volume

- ✓ **Finite-size effects have specific identifiable dependencies on size (L)**
- ✓ **The scaling of these dependencies give access to the CEP's location, it's critical exponents and scaling function → employed in this study**

Systematic studies of various quantities as function of √s are ongoing

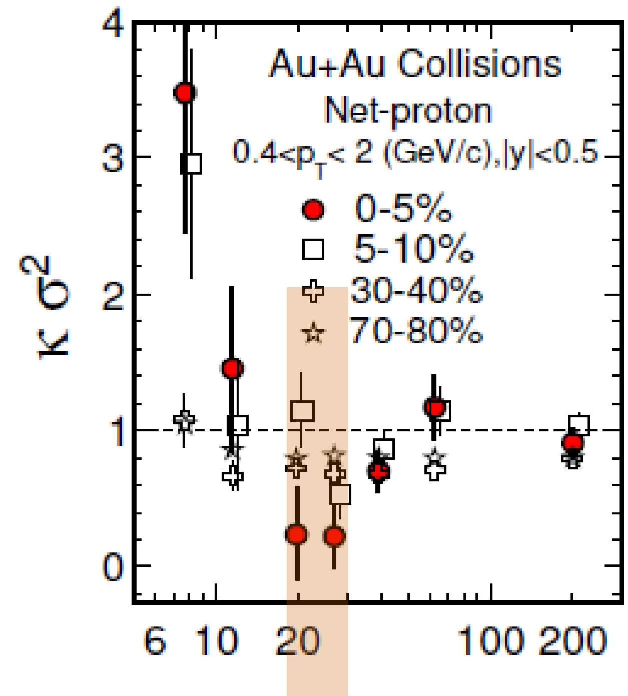
Good News

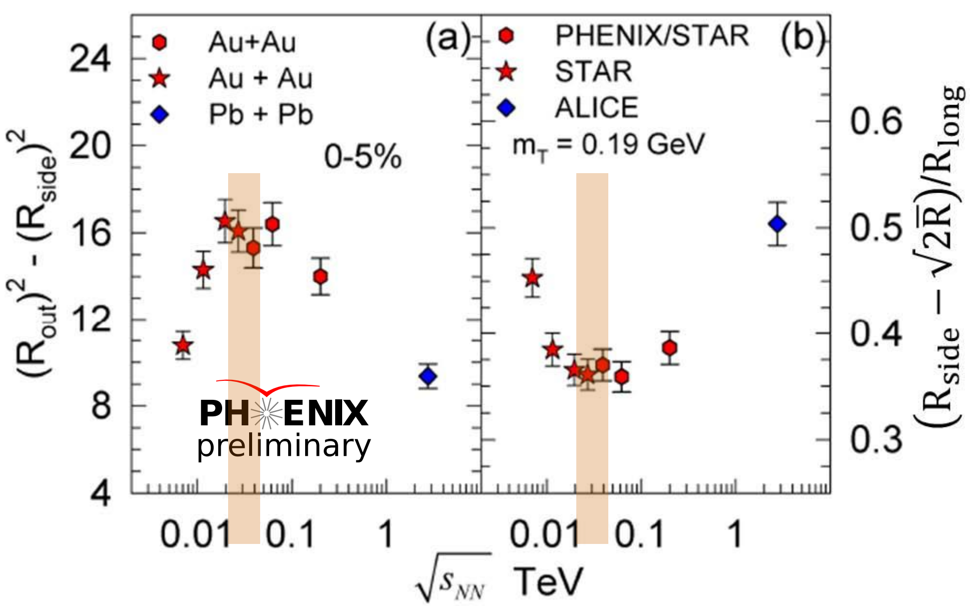
- **Several suggestive non-monotonic behavior at a ~ common √s**
  - ✓ **Focus on HBT probe**

Possible signals

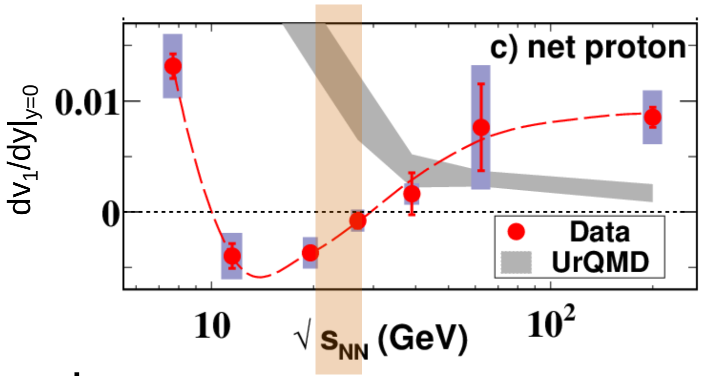
- Systematic study as function of $\sqrt{s}$:
  - Scaled kurtosis (baryon fluctuations)
  - Source radii

- **Suggestive non-monotonic behavior at a ~common $\sqrt{s}$**

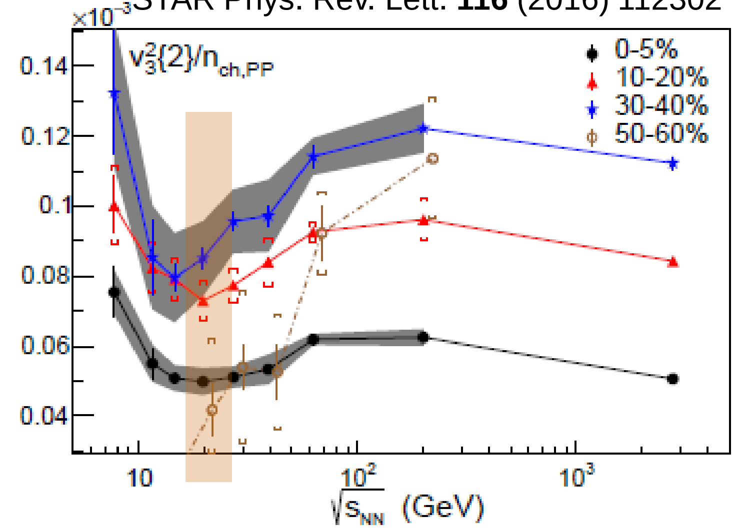




Possible signals



STAR Phys. Rev. Lett. **116** (2016) 112302

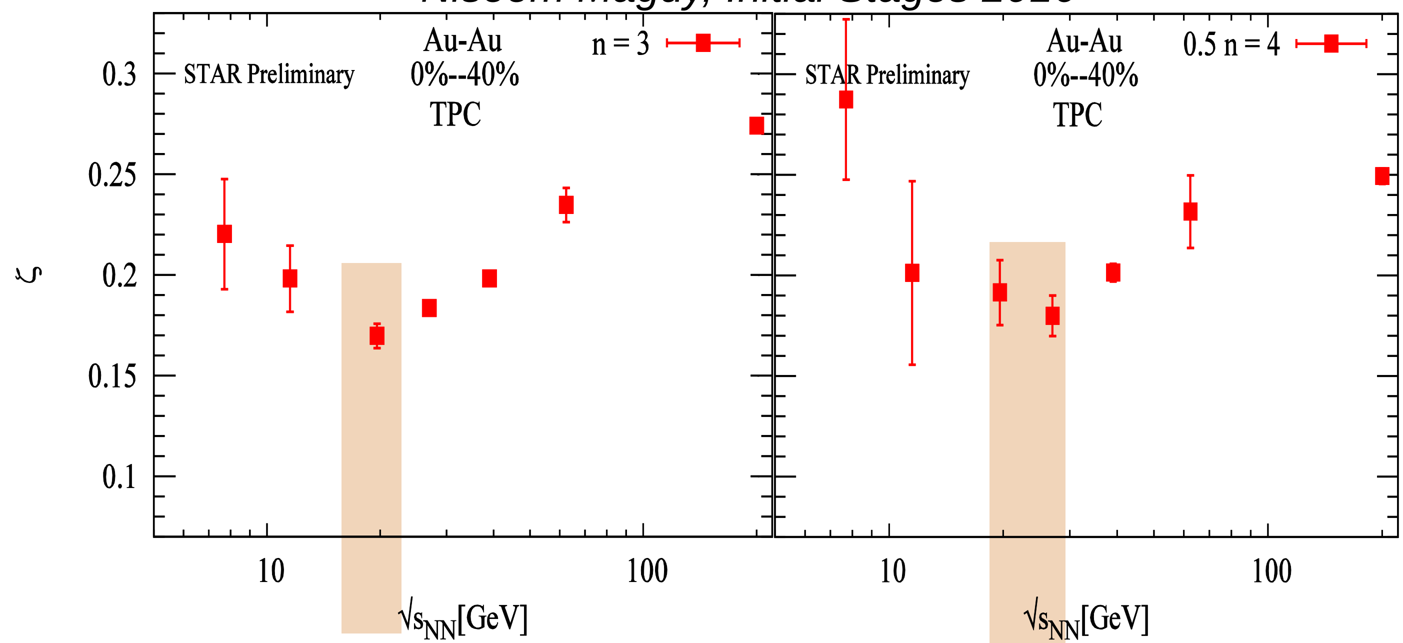


- Systematic study as function of $\sqrt{s}$:
  - v_n

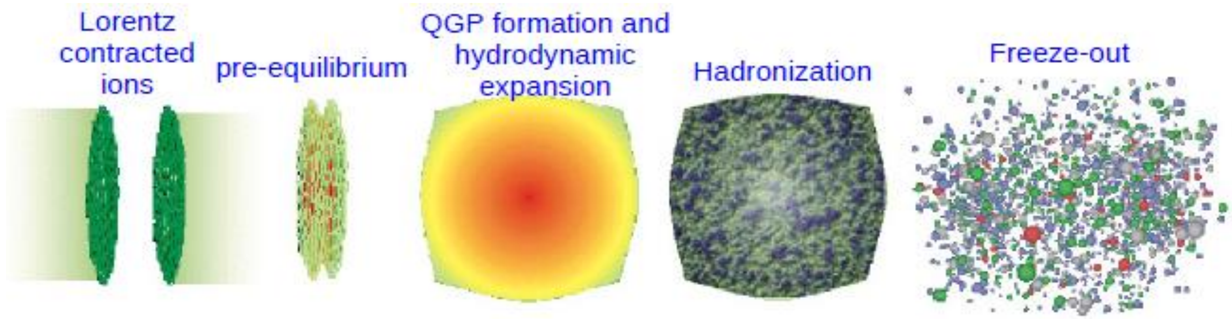
- **Suggestive non-monotonic behavior at a**

~ common $\sqrt{s}$

Niseem Magdy, Initial Stages 2016

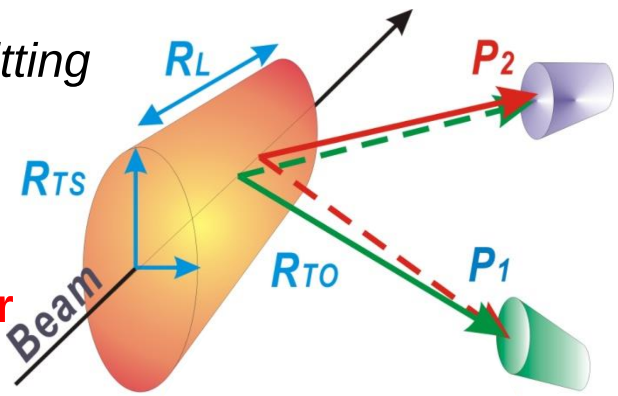


Interferometry as a susceptibility probe



The expansion of the emitting source (R_L, R_{To}, R_{Ts}) produced in HI collisions is driven by c_s

χ of the order parameter diverges at the CEP



$$c_s^2 = \frac{1}{\rho\kappa}$$

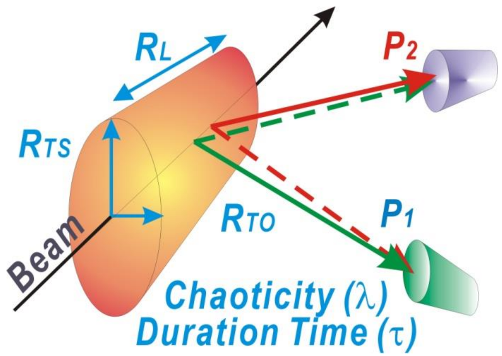
Susceptibility (χ)

In the vicinity of a phase transition or the CEP, the divergence of κ leads to anomalies in the expansion dynamics

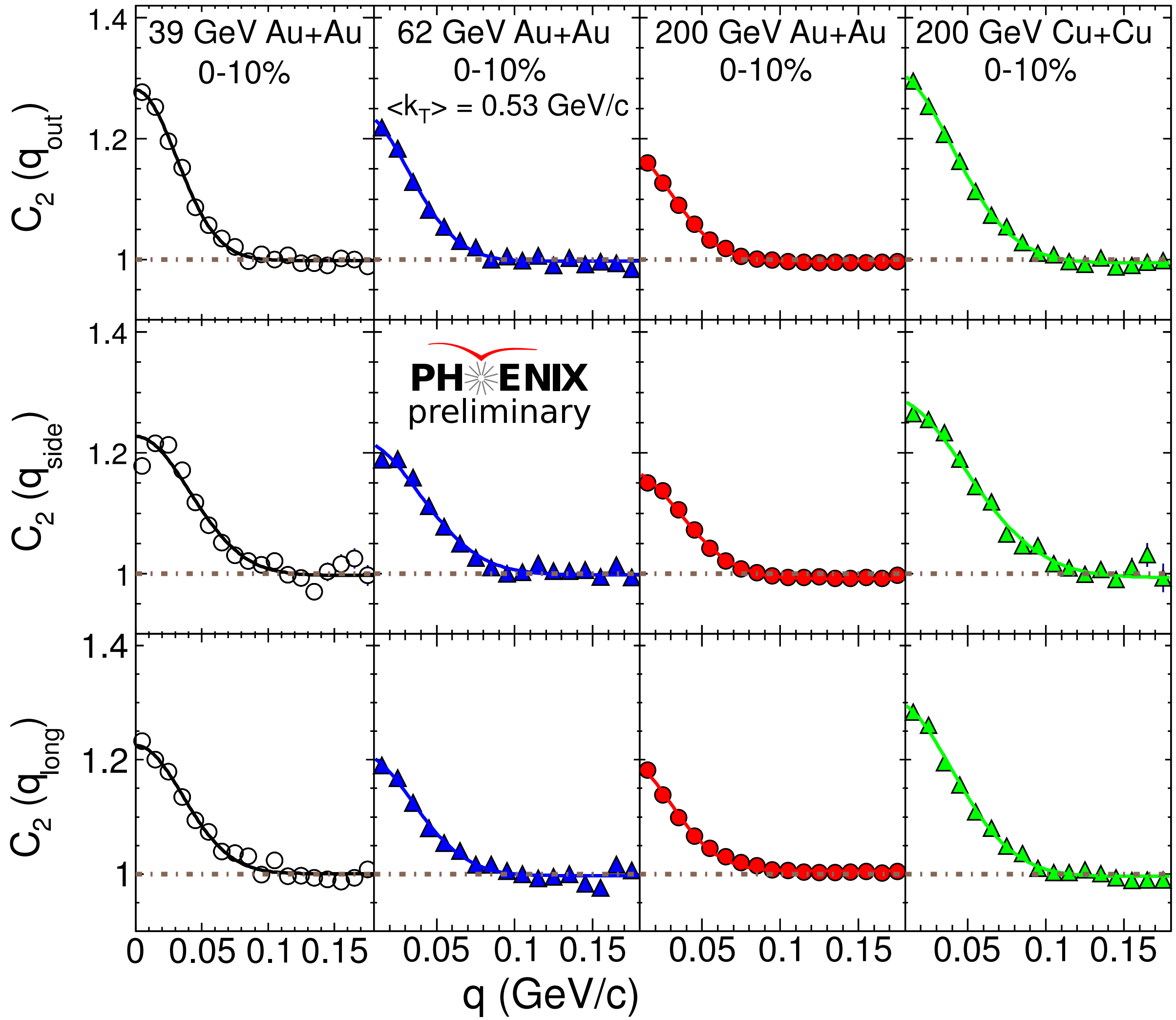
<u>Strategy</u>

Search for non-monotonic patterns for HBT radii combinations that are sensitive to the divergence of κ

Interferometry signal



$$C(\mathbf{q}) = \frac{dN_2 / d\mathbf{p}_1 d\mathbf{p}_2}{(dN_1 / d\mathbf{p}_1)(dN_1 / d\mathbf{p}_2)}$$



Adare et. al. (PHENIX)
arXiv:1410.2559

STAR -**Phys.Rev. C92 (2015) 1, 014904**

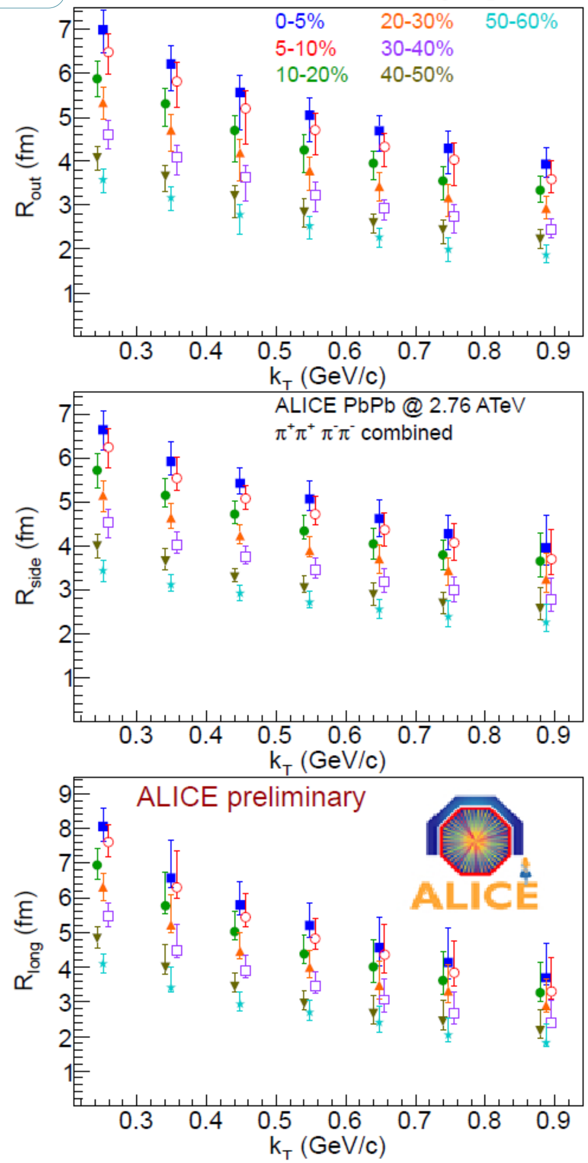
$$C_2(\mathbf{q}) = N[(\lambda(1 + G(\mathbf{q})))F_c + (1 - \lambda)],$$

$$G(\mathbf{q}) \cong \exp(-R^2_{\mathrm{side}}q^2_{\mathrm{side}} - R^2_{\mathrm{out}}q^2_{\mathrm{out}} - R^2_{\mathrm{long}}q^2_{\mathrm{long}})$$

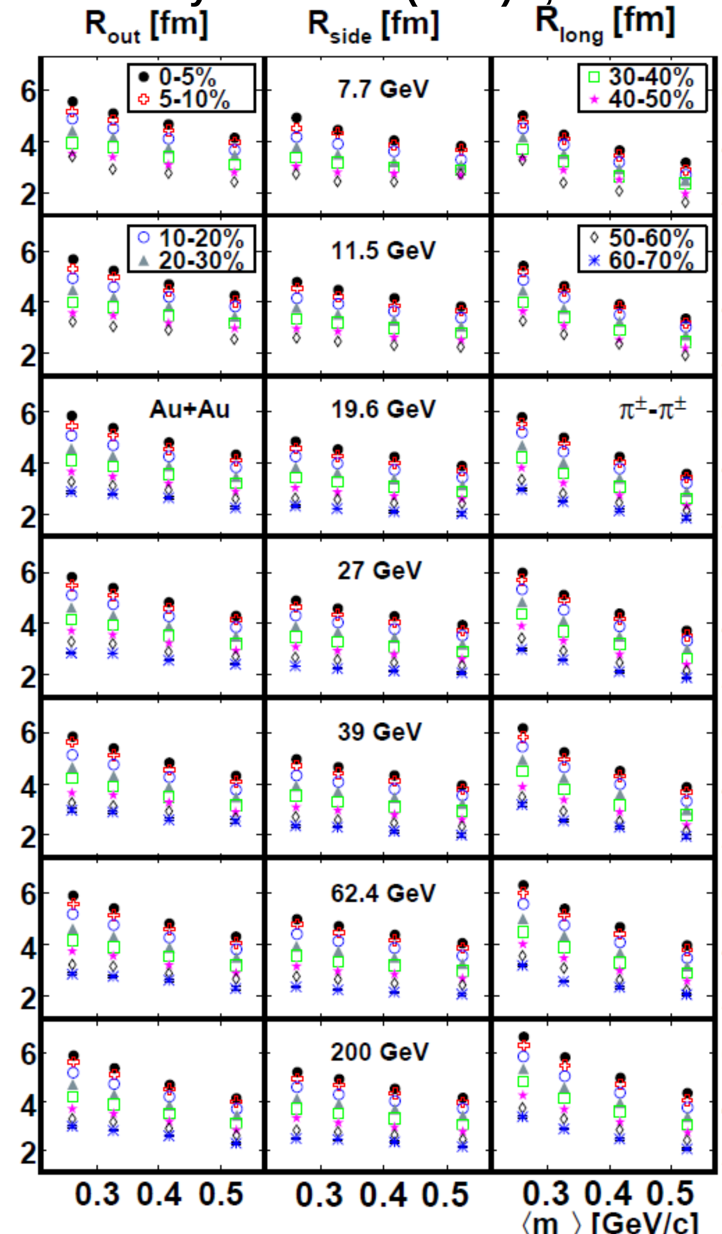
HBT Measurements

Phys. Rev. C 93 (2016) 024905

ALICE -PoS**WPCF2011**



STAR -**Phys.Rev. C92 (2015) 1, 014904**



This comprehensive set of two-pion HBT measurements is used in our search and characterization

***Strategy*: Search for non-monotonic patterns for HBT radii combinations that are sensitive to the divergence of κ**

Interferometry as a susceptibility probe

Hung, Shuryak, PRL. 75,4003 (95)
T. **Csörgő**. and B. Lörstad, PRC54 (1996) 1390-1403
Chapman, Scotto, Heinz, PRL.74.4400 (95)
Makhlin, Sinyukov, ZPC.39.69 (88)

$$R^2_{side} = \frac{R^2_{geo}}{1+\frac{m_T}{T}\beta_T^2}$$

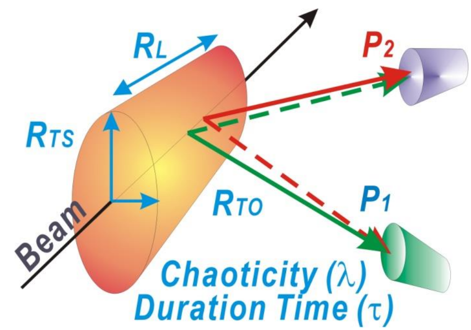
$$R^2_{out} = \frac{R^2_{geo}}{1+\frac{m_T}{T}\beta_T^2} + \beta_T^2(\Delta\tau)^2$$

emission duration

$$R^2_{long} \approx \frac{T}{m_T}\tau^2$$

emission lifetime

The measured HBT radii encode space-time information for the reaction dynamics



$$c_s^2 = \frac{1}{\rho\kappa}$$

The divergence of the susceptibility κ

- ✓ *"softens" the sound speed c_s*
- ✓ *extends the emission duration*

(R^2_{out} - R^2_{side}) sensitive to the κ

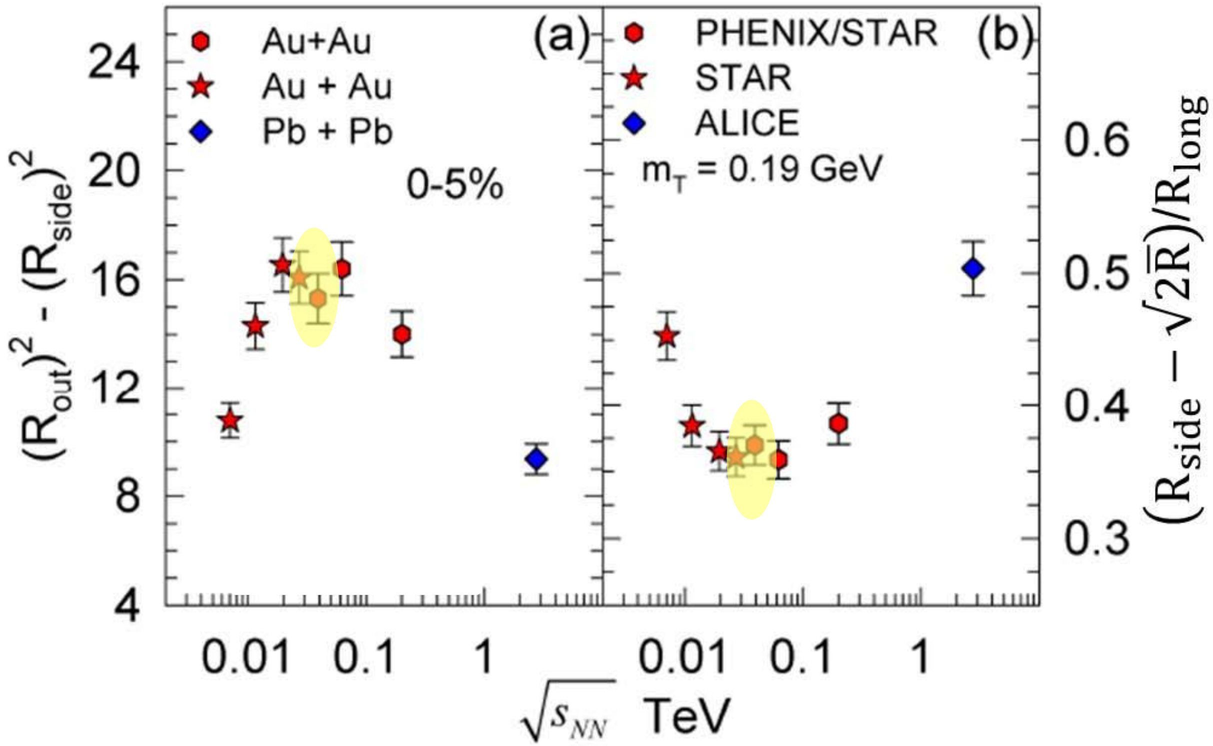
(R_{side} - R_{init})/ R_{long} sensitive to c_s

Specific non-monotonic patterns expected as a function of $\sqrt{s_{NN}}$

- ➢ **A maximum for (R^2_{out} - R^2_{side})**
- ➢ ***A minimum for* (R_{side} - $R_{initial}$)/R_{long}**

$\sqrt{s_{NN}}$ dependence of interferometry signal

Lacey QM2014
Adare et. al. (PHENIX)
arXiv:1410.2559



$R_{long} \propto \tau$

$\left(R_{out}^2 - R_{side}^2 \right) \propto \Delta\tau^2$

$\left(R_{side} - R_i \right) / R_{long} \propto u$

$R_{initial} = \sqrt{2}\,\bar{R}$

The measurements validate the expected non-monotonic patterns!

→ Reaction trajectories spend a fair amount of time near a "soft point" in the EOS that coincides with the CEP!

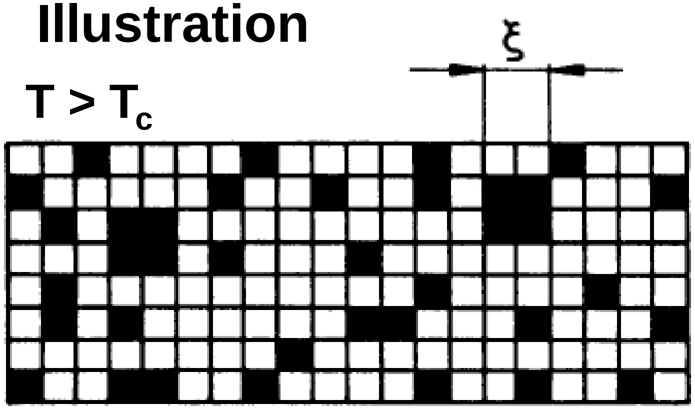
**** Note that R_{long}, R_{out} and R_{side} [all] increase with $\sqrt{s_{NN}}$ ****

<u>Finite-Size Scaling</u> (FSS) is used for further validation of the CEP, as well as to characterize its static and dynamic properties

Finite-Size Effects

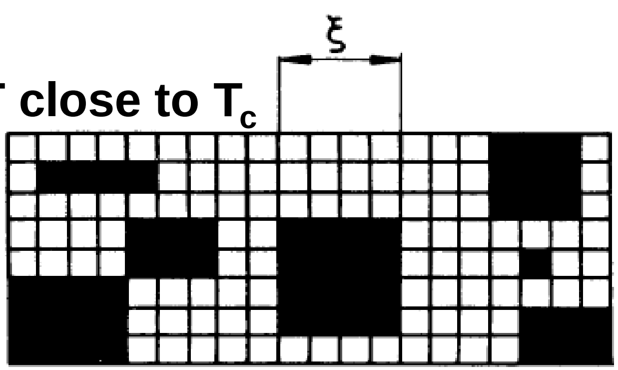
Illustration

T > T$_c$

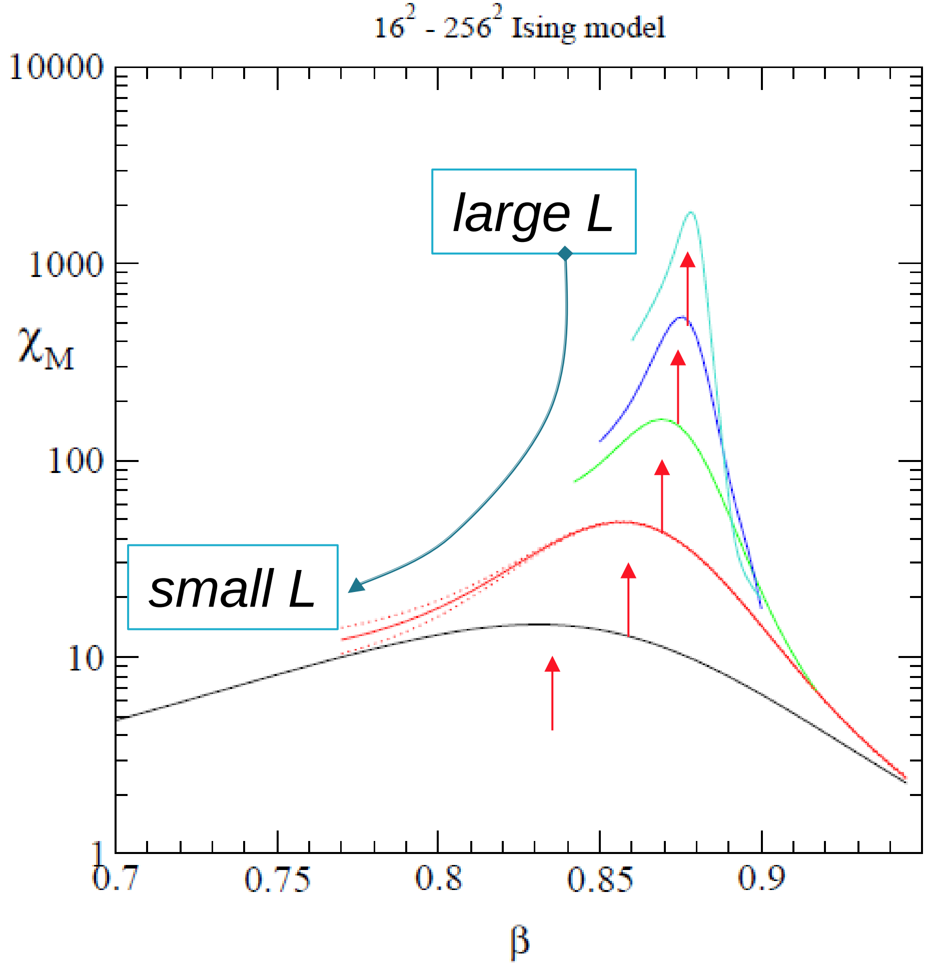


L characterizes the system size

T close to T$_c$



$$\xi \sim |T - T_c|^{-\nu} \le L$$

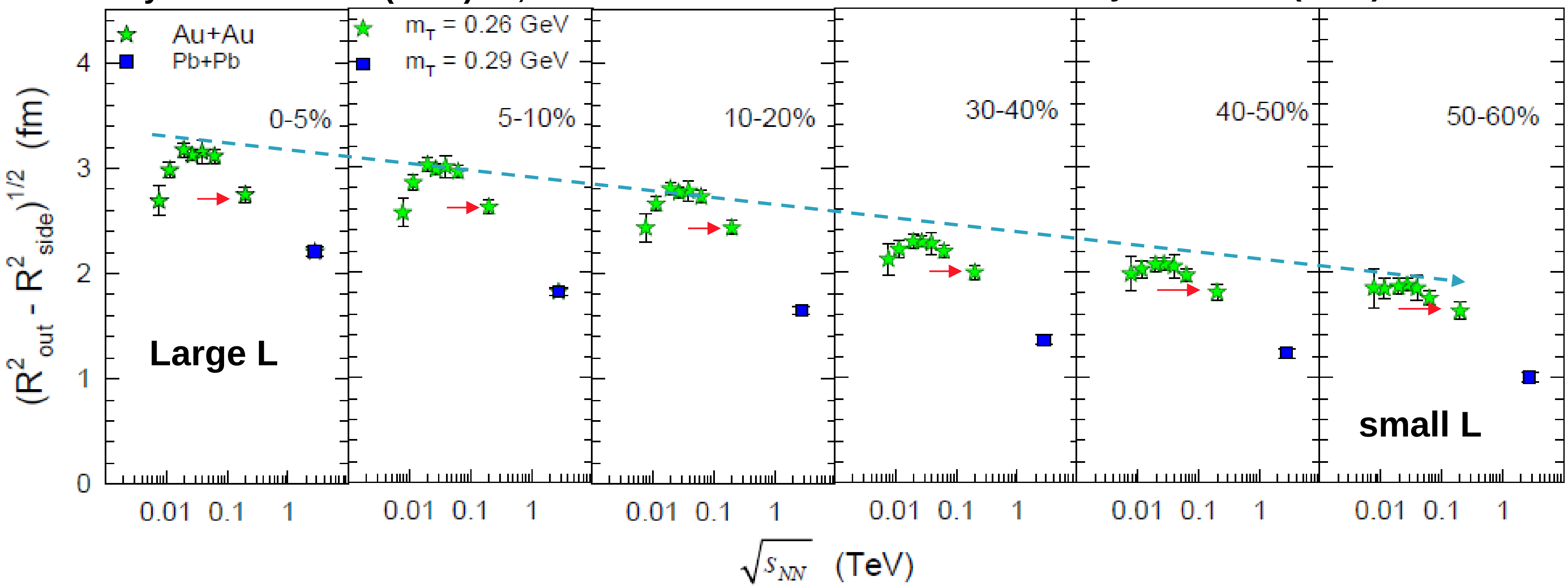


Note change in peak heights positions & widths with L

Size dependence of HBT excitation functions

Roy A. Lacey
Phys.Rev.Lett. 114 (2015) 14, 142301

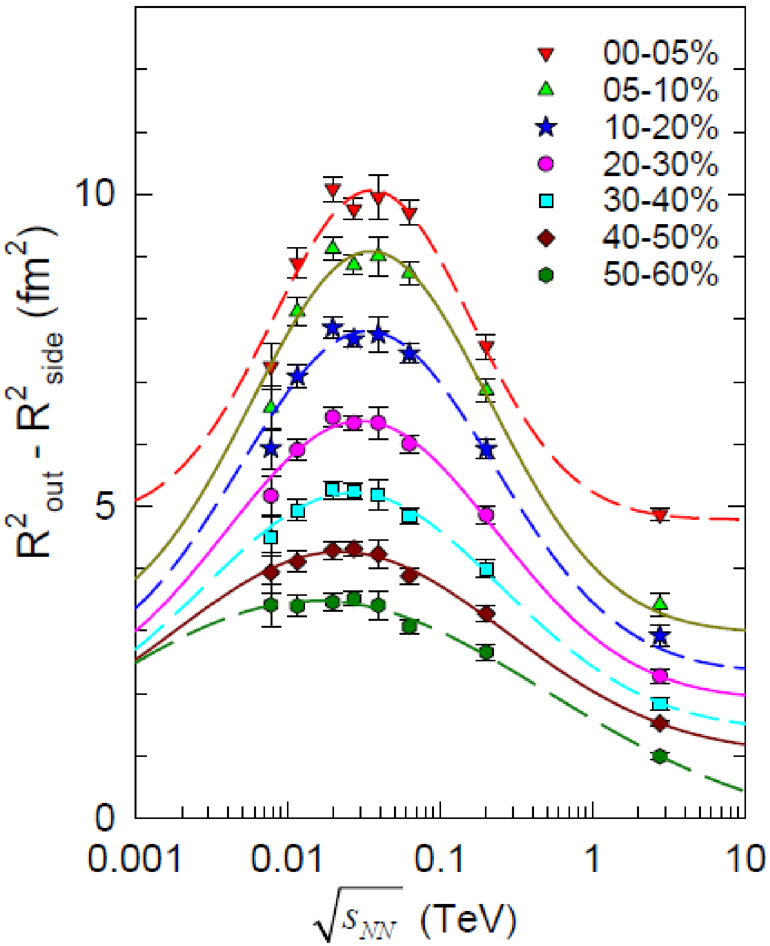
Data from L. Adamczyk et al. (STAR)
Phys.Rev. C92 (2015) 1, 014904

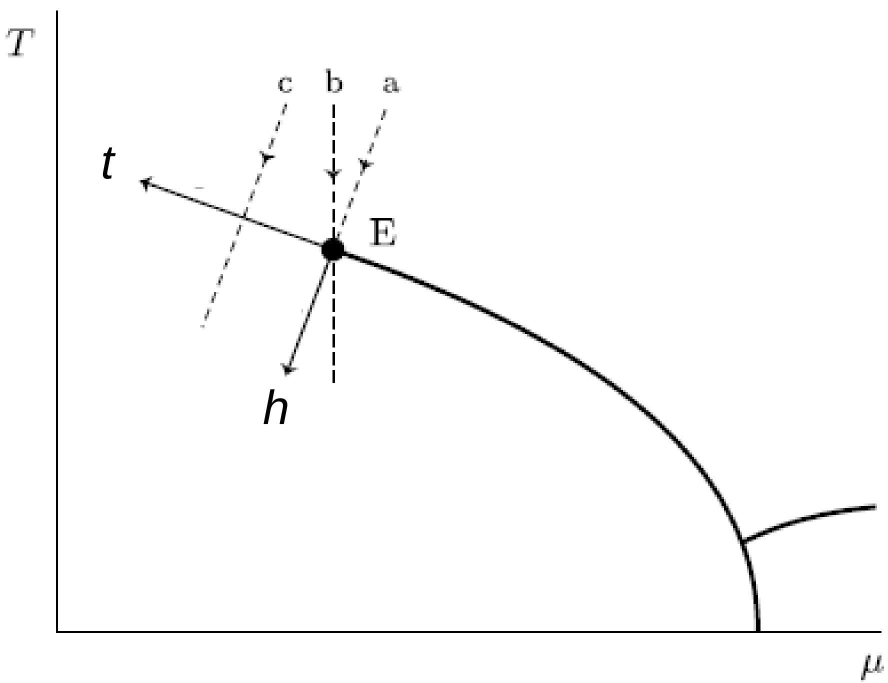


The data validate the expected patterns for Finite-Size Effects

- ✓ **Max values decrease with decreasing system size**
- ✓ **Peak positions shift with decreasing system size**
- ✓ **Widths increase with decreasing system size**

Size dependence of HBT excitation functions





characteristic patterns signal the effects of finite-size

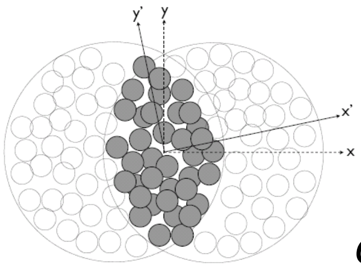
I. Use (R^2_{out}-R^2_{side}) as a proxy for the susceptibility

II. Parameterize distance to the CEP by $\sqrt{s_{NN}}$

$$\tau_s = \left(\sqrt{s} - \sqrt{s_{CEP}}\right)/\sqrt{s_{CEP}}$$

III. Perform Finite-Size Scaling analysis with length scale $L = \bar{R}$

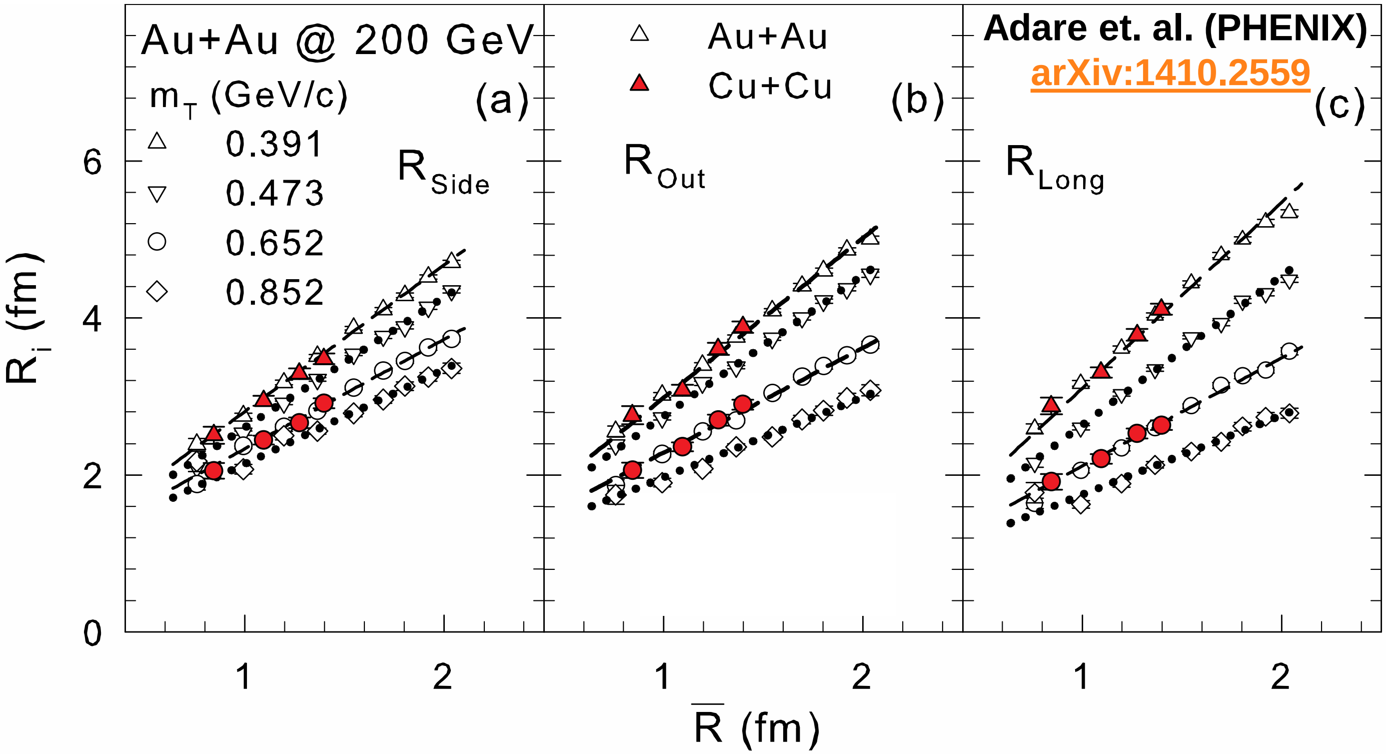
Length Scale for Finite Size Scaling

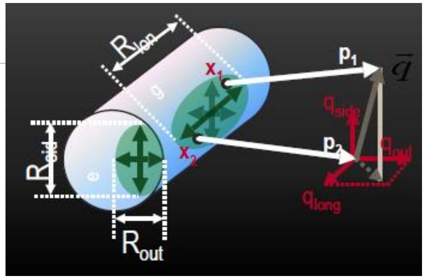


$$\frac{1}{\bar{R}} = \sqrt{\left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}\right)}$$

$\bar{R}$ is a characteristic length scale of the initial-state transverse size,

σ_x & σ_y → RMS widths of density distribution





$$R_{out}, R_{side}, R_{long} \propto \bar{R}$$

$\bar{R}$ scales the volume

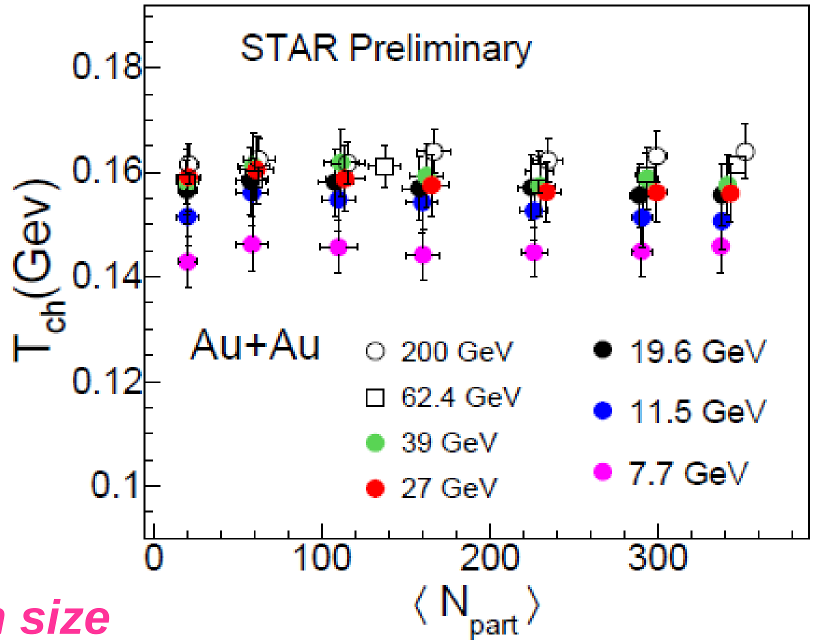
➤ ***$\bar{R}$ scales the full RHIC and LHC data sets***

Summary of Scaling Procedure

(only two exponents are independent)

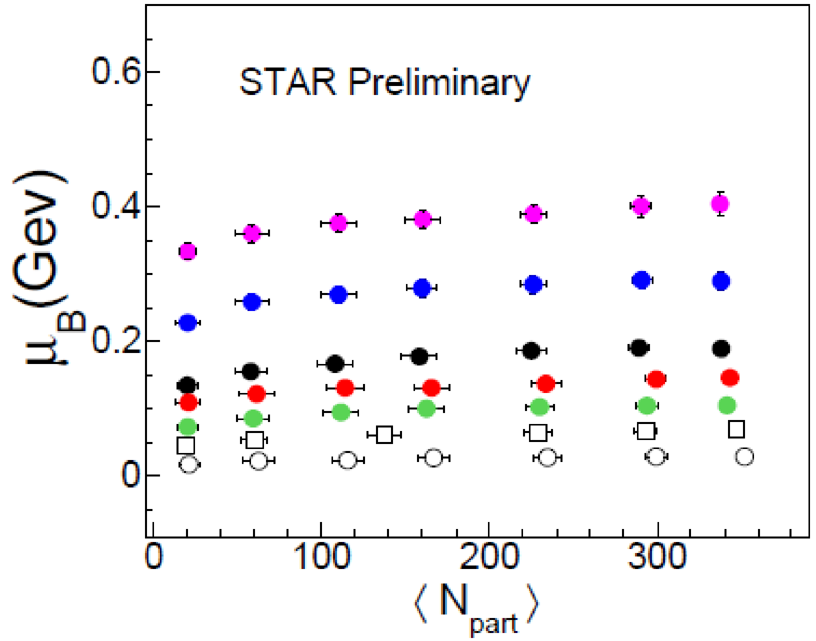
$$\chi_T^{\max}(V) \sim L^{\gamma/\nu},$$
$$\delta T(V) \sim L^{-\frac{1}{\nu}},$$
$$\tau_T(V) \sim T^{\text{cep}}(V) - T^{\text{cep}}(\infty) \sim L^{-\frac{1}{\nu}},$$

$$\boxed{\begin{aligned}(R^2_{\text{out}} - R^2_{\text{side}})^{\max} &\propto \bar{R}^{\gamma/\nu}, \\ \sqrt{s_{NN}}(V) &= \sqrt{s_{NN}}(\infty) - k \times \bar{R}^{-\frac{1}{\nu}},\end{aligned}}$$



Note that (μ_B^f, T^f) *is not strongly dependent on size*

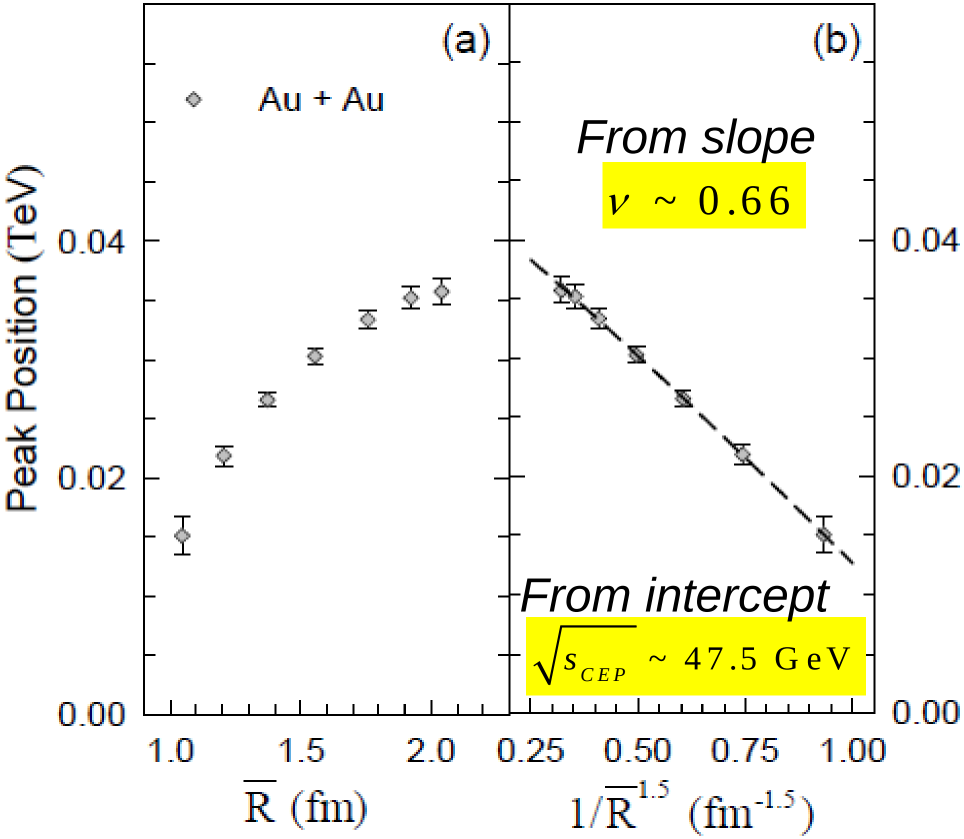
- ***Extract position ($\sqrt{s_{NN}}$) of deconfinement transition and critical exponents***
- ***Use exponents to determine:***
  - ✓ ***Order of the phase transition***
  - ✓ ***Static universality class***

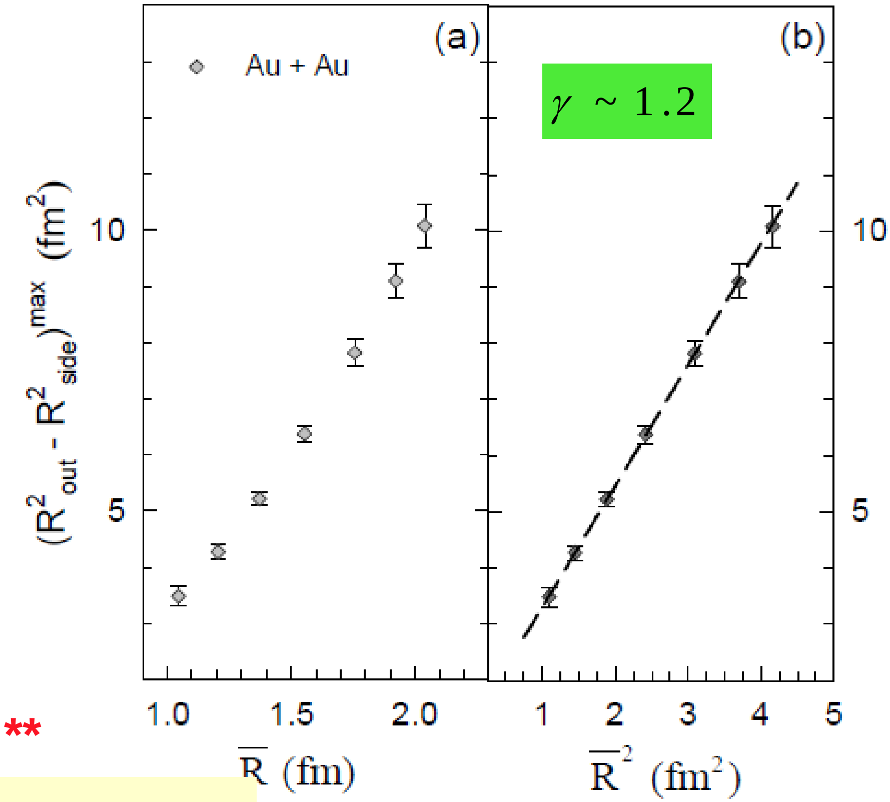


Finite – Size Scaling

$$\sqrt{s_{NN}}(V) = \sqrt{s_{NN}}(\infty) - k \times \overline{R}^{-1/\nu}$$

$$\left(R^2_{out} - R^2_{side}\right)^{max} \propto \overline{R}^{\gamma/\nu}$$





**** Same ν value from analysis of the widths ****

- ***The critical exponents validate***
  - ✓ ***the 3D Ising model (static) universality class***
  - ✓ ***2^{nd} order phase transition for CEP***

$$T^{cep} \sim 165 \text{ MeV}, \mu_B^{cep} \sim 95 \text{ MeV}$$

($\sqrt{s_{CEP}}$ & chemical freeze-out systematics)

Closurer test for FSS

- ***2^{nd} order phase transition***
- ***3D Ising Model (static) universality class for CEP***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

$\chi(T, L) = L^{\gamma/\nu} P_\chi(tL^{1/\nu})$

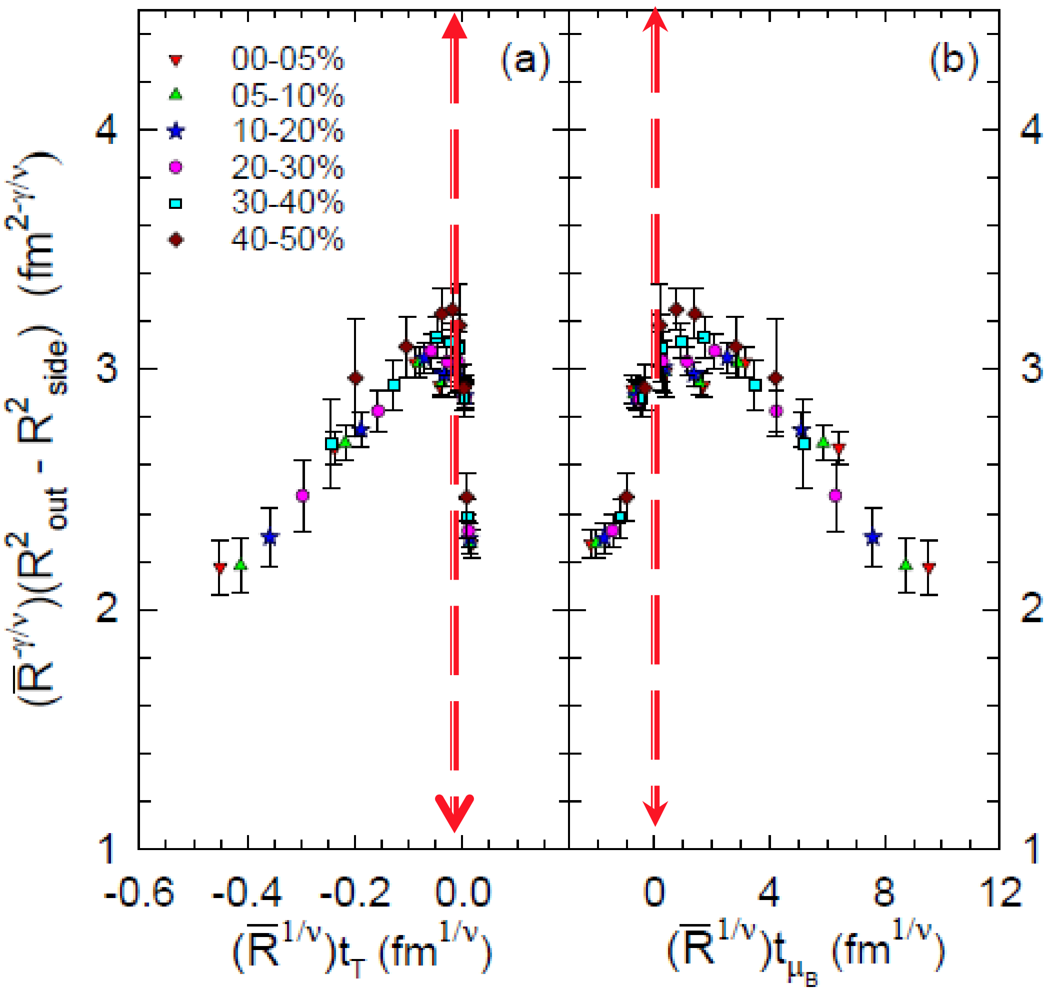
M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

Use T^{cep}, μ_B^{cep}, ν and γ to obtain Scaling Function P_χ

$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_T,$$
$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_{\mu_B},$$

$$t_T = (T - T^{\rm cep})/T^{\rm cep}$$
$$t_{\mu_B} = (\mu_B - \mu_B^{\rm cep})/\mu_B^{\rm cep}$$

T anf μ_B *are* from $\sqrt{s_{NN}}$



*****A further validation of the location of the CEP and the (static) critical exponents*****

Dynamic Finite – Size Scaling

- ***2nd order phase transition***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165\ \mathrm{MeV},\ \mu_B^{cep} \sim 95\ \mathrm{MeV}$

DFSS ansatz

at time τ when T is near T_{cep}

$$\chi(L,T,\tau) = L^{\gamma/\nu} f\left(L^{1/\nu} t_T, \tau L^{-z}\right)$$

$$t_T = (T - T^{\mathrm{cep}})/T^{\mathrm{cep}}$$

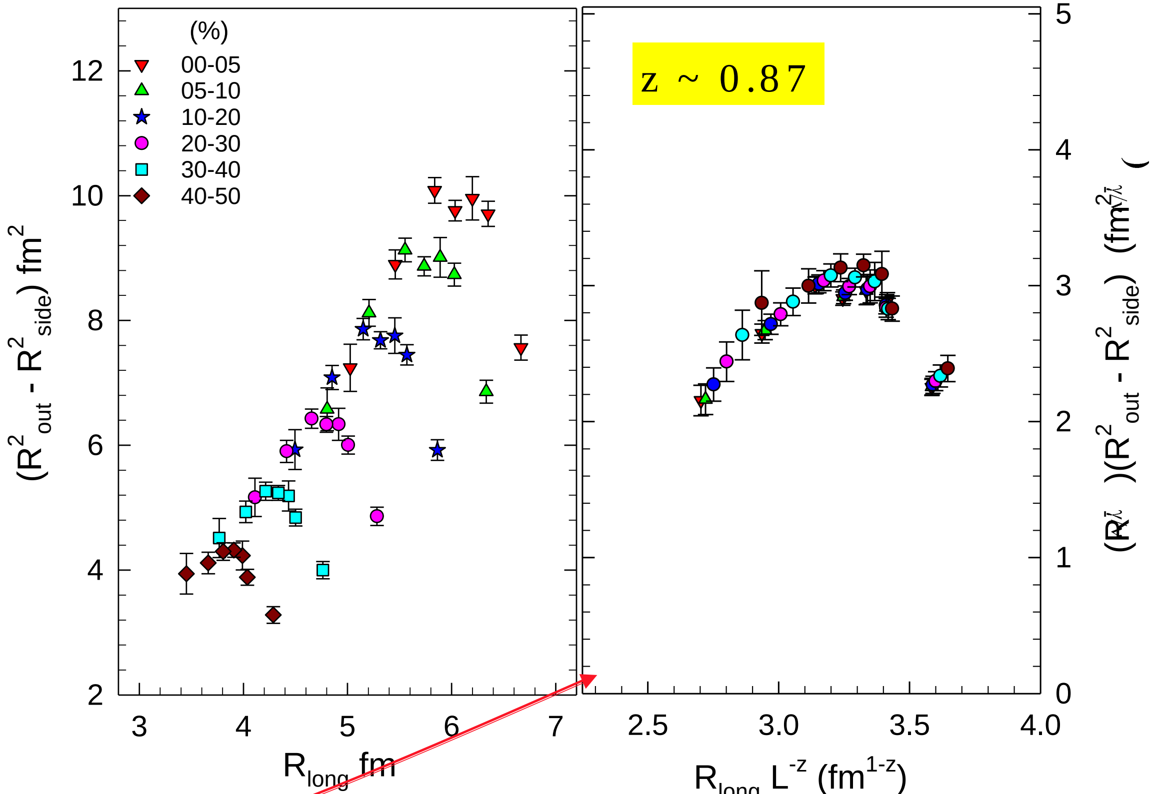
For
$T = T_c$

$$\chi(L,T_c,\tau) = L^{\gamma/\nu} f\left(\tau L^{-z}\right)$$

$R_{long} \propto \tau$

M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

*****Experimental estimate of the dynamic critical exponent*****



The magnitude of z is similar to the predicted value for z_s but the sign is opposite

Epilogue

Strong experimental indication for the CEP and its location

(Dynamic) Finite-Size Scalig analysis

- ***3D Ising Model (static) universality class for CEP***
- ***2nd order phase transition***

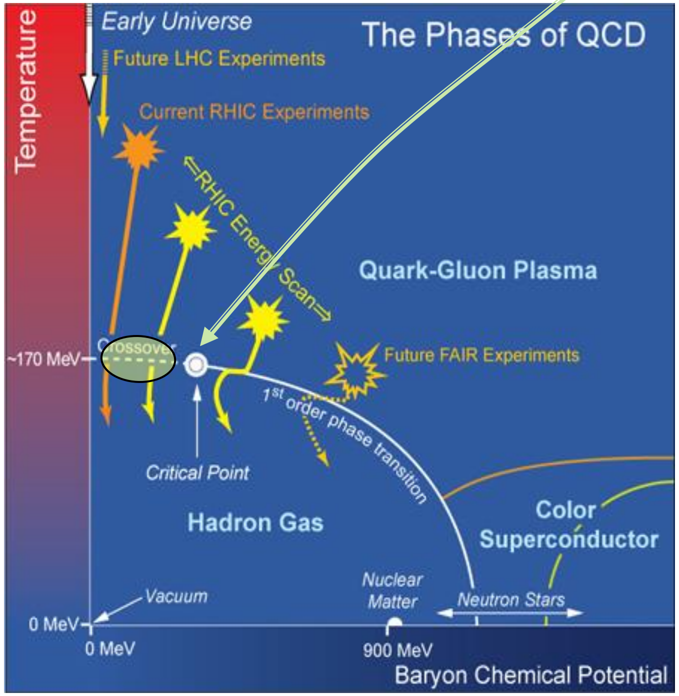
$T^{cep} \sim 165\ \mathrm{MeV},\ \mu_B^{cep} \sim 95\ \mathrm{MeV}$

$\nu \sim 0.66$

$\gamma \sim 1.2$

$z \sim 0.87$

- ✓ *Landmark validated*
- ✓ *Crossover validated*
- ✓ *Deconfinement validated*
- ✓ *(Static) Universality class validated*
- ✓ *Model H dynamic Universality class invalidated?*
- ✓ *Other implications!*



New Data from RHIC (BES-II) together with theoretical modeling, can provide crucial validation tests for the coexistence regions, as well as to firm-up characterization of the CEP!

Much additional work required to get to "the end of the line"

End

Phys.Rev.Lett.100:232301,2008)

Source breakup dynamics in Au+Au Collisions at $\sqrt{s_{NN}}$=200 GeV via three-dimensional two-pion source imaging

S. Afanasiev,[17] C. Aidala,[7] N.N. Ajitanand,[43] Y. Akiba,[37,38] J. Alexander,[43] A. Al-Jamel,[33] K. Aoki,[23,37] L. Aphecetche,[45] R. Armendariz,[33] S.H. Aronson,[3] R. Averbeck,[44] T.C. Awes,[34] B. Azmoun,[3] V. Babintsev,[14] A. Baldisseri,[8] K.N. Barish,[4] P.D. Barnes,[26] B. Bassalleck,[32] S. Bathe,[4] S. Batsouli,[7] V. Baublis,[36] F. Bauer,[4] A. Bazilevsky,[3] S. Belikov,[3,16,*] R. Bennett,[44] Y. Berdnikov,[40] M.T. Bjorndal,[7] J.G. Boissevain,[26] H. Borel,[8]

Phys.Lett. B685 (2010) 41-46

Three-dimensional two-pion source image from Pb+Pb collisions at $\sqrt{s_{NN}}$=17.3 GeV: new constraints for source breakup dynamics

C. Alt[9], T. Anticic[23], B. Baatar[8],D. Barna[4], J. Bartke[6], L. Betev[10], H. Białkowska[20], C. Blume[9], B. Boimska[20], M. Botje[1], J. Bracinik[3], P. Bunčić[10], V. Cerny[3], P. Christakoglou[1], P. Chung[19], O. Chvala[14], J.G. Cramer[16], P. Csató[4], P. Dinkelaker[9], V. Eckardt[13], D. Flierl[9], Z. Fodor[4], P. Foka[7], V. Friese[7], J. Gál[4], M. Gaździcki[9,11], V. Genchev[18], E. Gładysz[6], K. Grebieszkow[22], S. Hegyi[4], C. Höhne[7], K. Kadija[23], A. Karev[13], S. Kniege[9], V.I. Kolesnikov[8], R. Korus[11], M. Kowalski[6], M. Kreps[3], A. Laszlo[4], R. Lacey[19], M. van Leeuwen[1], P. Lévai[4], L. Litov[17], B. Lungwitz[9], M. Makariev[17], A.I. Malakhov[8], M. Mateev[17], G.L. Melkumov[8],

$$\tau = \tau_0 + a\rho$$

Space-time correlation parameter

Interferometry as a susceptibility probe

Dirk Rischke and Miklos Gyulassy
Nucl.Phys.A608:479-512,1996

Fig. 1

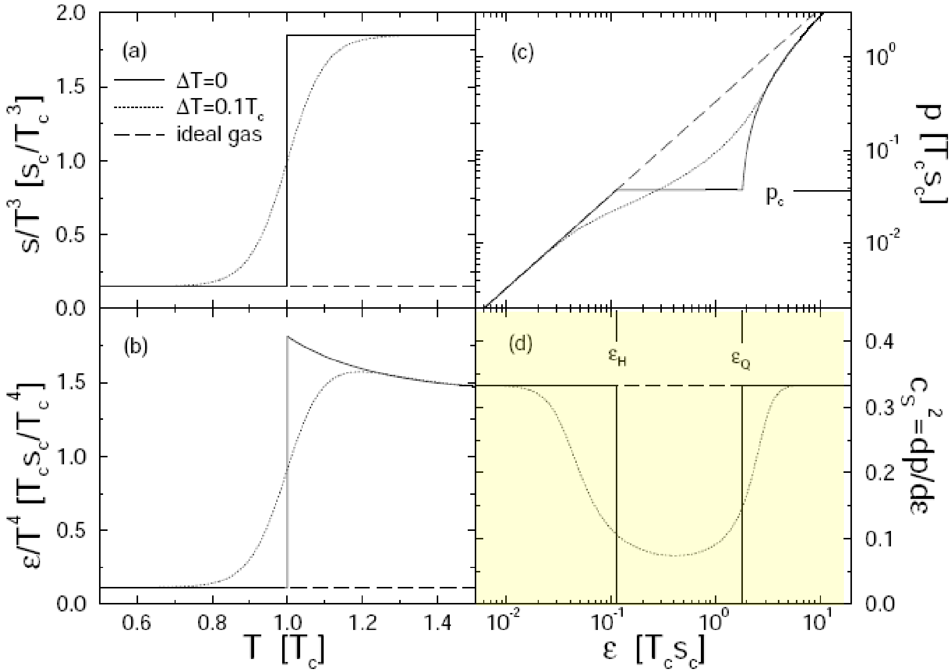
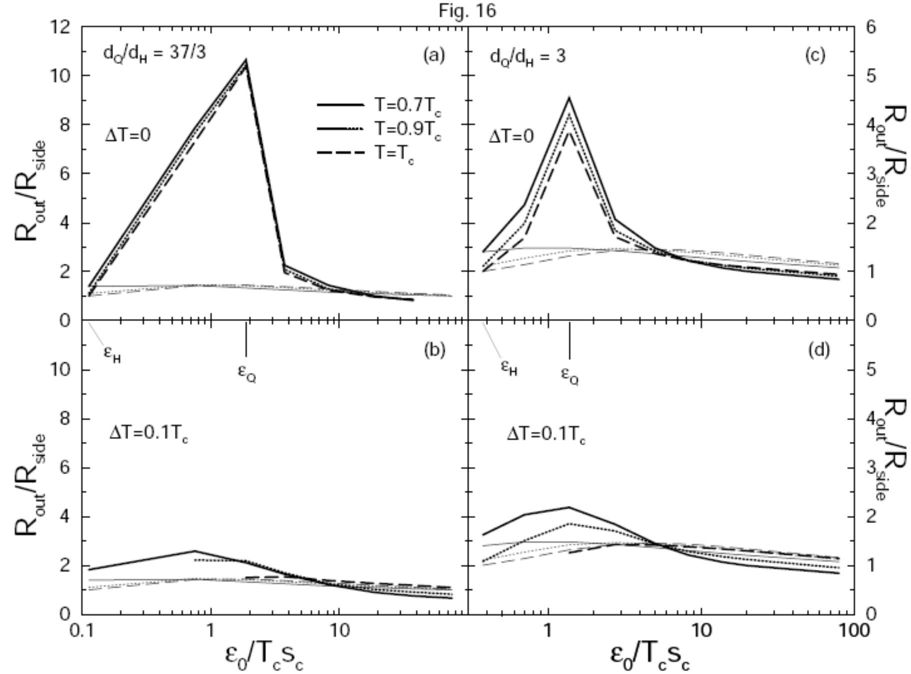


Fig. 16



In the vicinity of a phase transition or the CEP, the sound speed is expected to soften considerably.

$$c_s^2 = \frac{1}{\rho \kappa_S}$$

Divergence of the compressibility (κ)

→ non-monotonic excitation function for (R^2_{out} - R^2_{side}) *due to an enhanced emission duration*

Theoretical Guidance

Theory consensus on the static universality class for the CEP

3D-Ising Z(2)

- ✓ $\nu \sim 0.63$
- ✓ $\gamma \sim 1.2$

M. A. Stephanov
Int. J. Mod. Phys. A 20, 4387 (2005)

Dynamic Universality class for the CEP less clear

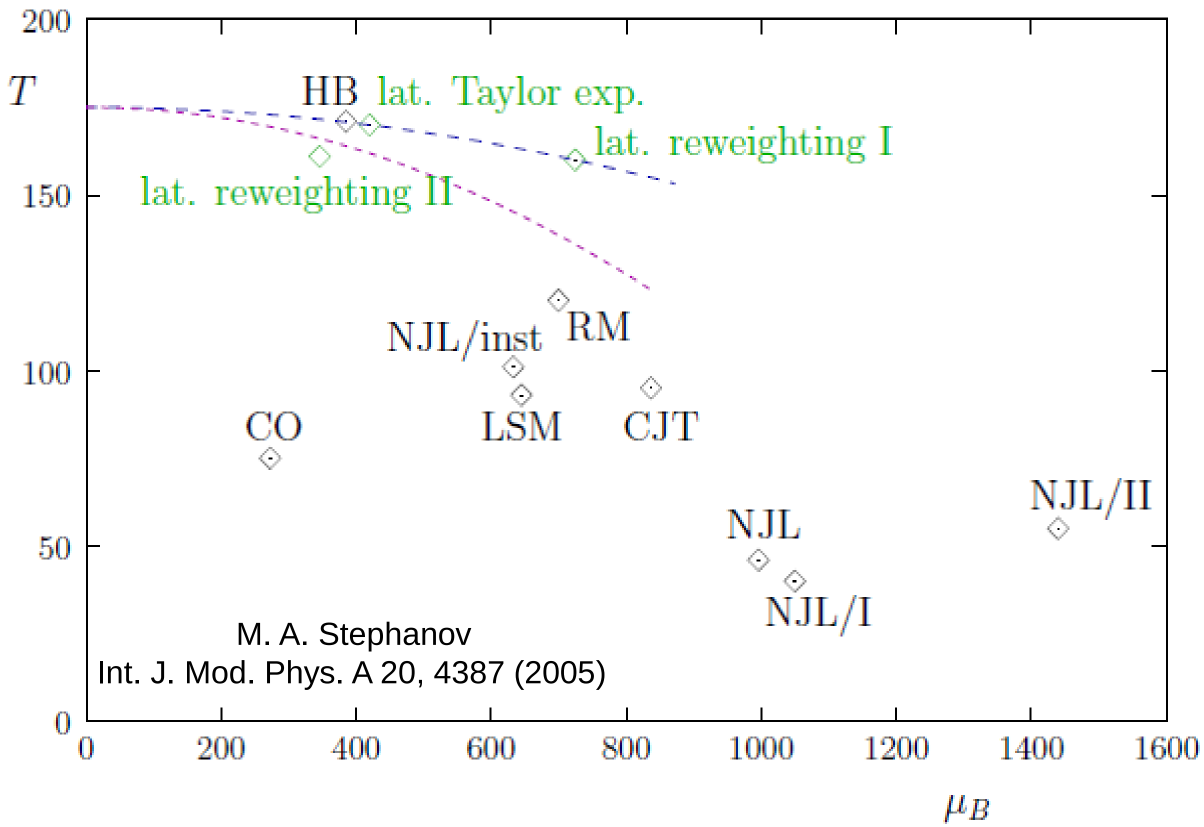
- **One slow mode (L)**
  - ✓ ***z ~ 3 - Model H***

Son & Stephanov
Phys.Rev. D70 (2004) 056001
Moore & Saremi ,
JHEP 0809, 015 (2008)

- **Three slow modes (NL)**
  - ✓ $z_T \sim 3$
  - ✓ $z_v \sim 2$
  - ✓ $z_s \sim -0.8$

Y. Minami - **Phys.Rev. D83 (2011) 094019**

The predicted location (T^{cep}, μ_B^{cep}) of the CEP is even less clear!



Experimental verification and characterization of the CEP is a crucial ingredient

What about Finite-Time Effects (FTE)?

χ_{op} diverges at the CEP
so relaxation of the order parameter could be anomalously slow

$\tau \sim \xi^{z}$ ← dynamic critical exponent

z > 0 - Critical slowing down

Non-linear dynamics → Multiple slow modes

*$z_T \sim 3$, $z_v \sim 2$, **$z_s \sim -0.8$***

$z_s < 0$ - Critical speeding up

Y. Minami - **Phys.Rev. D83 (2011) 094019**

An important consequence

$$\xi \sim \tau^{1/z}$$

Significant signal attenuation for short-lived processes with $z_T \sim 3$ or $z_v \sim 2$

eg. $\langle(\delta n)\rangle \sim \xi^2$ **(without FTE)**

$\langle(\delta n)\rangle \sim \tau^{1/z} \ll \xi^2$ **(with FTE)**

The value of the dynamic critical exponent/s is crucial for HIC

Dynamic Finite-Size Scaling (DFSS) is used to estimate the dynamic critical exponent z